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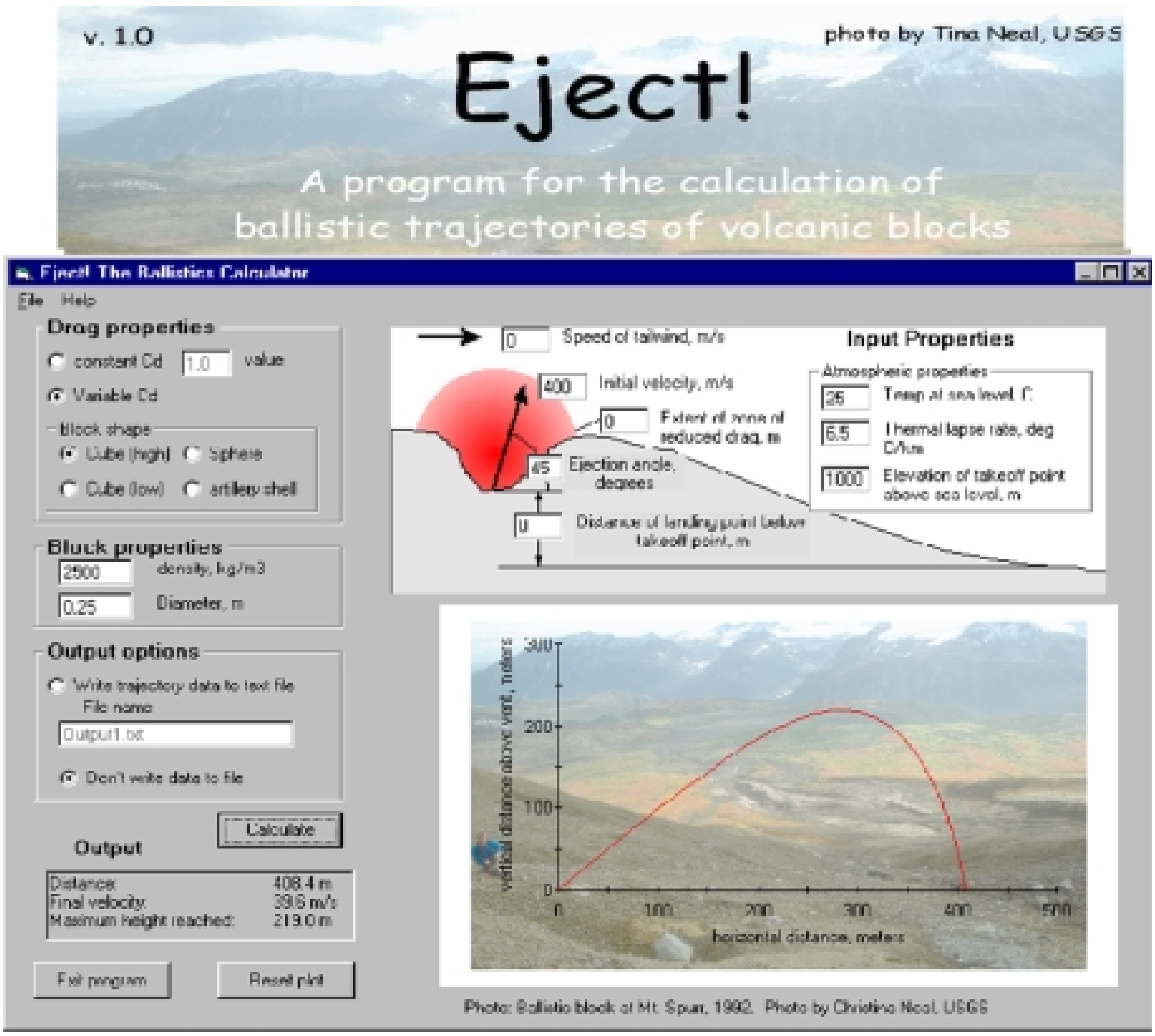
A Simple Calculator of Ballistic Trajectories for Blocks Ejected during Volcanic Eruptions



U.S.Geological Survey Open-File Report 01-45

U.S. Department of the Interior
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Symbol	Explanation	Units
A	cross-sectional area of block	m^2
C_d	drag coefficient	—
C_p	specific heat of gas at constant pressure	J/(kg K)
C_v	specific heat of gas at constant volume	J/(kg K)
c	speed of sound	m/s
D	block diameter	m
g	gravitational acceleration	m/s^2
M	Mach number	—
m	mass of block	kg
R	gas constant for air	J/(kg K)
Re	Reynolds number	—
r_o	radius from vent at which blocks begin to decelerate	m
r_d	radius from vent over which drag is reduced	m
t	time since ejection	s
t_o	time since ejection at which deceleration begins	s
v	magnitude of velocity vector	m/s
v_x	velocity component in x direction	m/s
v_z	velocity component in z direction	m/s
w	tailwind velocity	m/s
x	distance from vent in horizontal direction	m
z	vertical distance above sea level	m
z_{vent}	elevation of vent above sea level	m
η	viscosity of air	Pa s
γ	ratio of specific heats (C_p/C_v) of air	—
μ	$= (\rho_a C_d A)/2m$	kg/m^2
ρ_a	air density	kg/m
ρ_r	block density	kg/m^3
θ	angle of block ejection from horizontal	radians

A Simple Calculator of Ballistic Trajectories for Blocks Ejected During Volcanic Eruptions

By **Larry G. Mastin**

INTRODUCTION

During the past century, numerous observers have described the violent ejection of large blocks and bombs from volcanoes during volcanic explosions. Minakami (1942) mapped the locations of blocks ejected from Asama Volcano during explosions in 1937. He developed a mathematical expression relating initial velocity and trajectory angle of ejected blocks to the ejection distance, taking into account air drag and assuming a constant drag coefficient. In the late 1950's, Gorshkov (1959) estimated ejection velocities at Bezymianny volcano during its sector-collapse eruption. Wilson (1972) developed the first mathematical algorithm for ballistic trajectories in the volcanological literature (earlier ones had been available for military applications) that considered variations in drag coefficient with Reynolds number. Fagents and Wilson (1993) advanced the method of Wilson (1972) by considering the effect of reduced drag near the vent. From the 1970's through the 1990's other papers, too numerous to mention, have estimated volcanic ejection velocities from ballistic blocks.

Since the early 1990's there has been a decrease in the number of published papers that quantify ejection velocities from ballistic trajectories. This decrease has resulted in part from the appreciation that ejection velocities cannot be uniquely determined by ejection distance due to uncertainties in initial trajectory angle and drag force. On the other hand, the decrease in usage has coincided with an increase in the ease with which ballistic calculations can be made, due to the vast improvement in computer power and in the user-friendliness of computers. During the 1970's, only volcanologists with mathematical acumen or those who could collaborate with applied mathematicians were able to make such estimates. With 21st century computer power, ballistic computation should be available to anyone as a back-of-the-envelope indicator of explosive power; the only factor preventing such usage is the lack of a user-friendly computer program.

In this paper, I describe a program that can be used for quick ballistics calculations. The program, Eject!, was written in Microsoft Visual Basic® and operates on any personal computer running Microsoft® Windows 95 or later. The executable file, source code, and documentation that you are reading have been posted on the Internet for easy access. Below is an explanation of the physics and numerical method behind the calculations.

METHOD

During volcanic eruptions, discrete explosions (e.g., fig. 1) eject clouds of ash, lapilli, blocks and bombs, volcanic gas, and, in some circumstances, external water. Within seconds after ejection, the cloud front generally evolves from a more-or-less spherical form in into one consisting of finger-like projections ("finger jets"); at the apex of each finger is a single ballistic block or a collection of larger fragments. Behind the finger front is a cloud of smaller tephra fragments and gas that ride within the slipstream of the larger blocks. Seconds after the front of ballistic fingers becomes perceptible, blocks may completely separate from the trailing cloud and travel on their own through the ambient atmosphere.

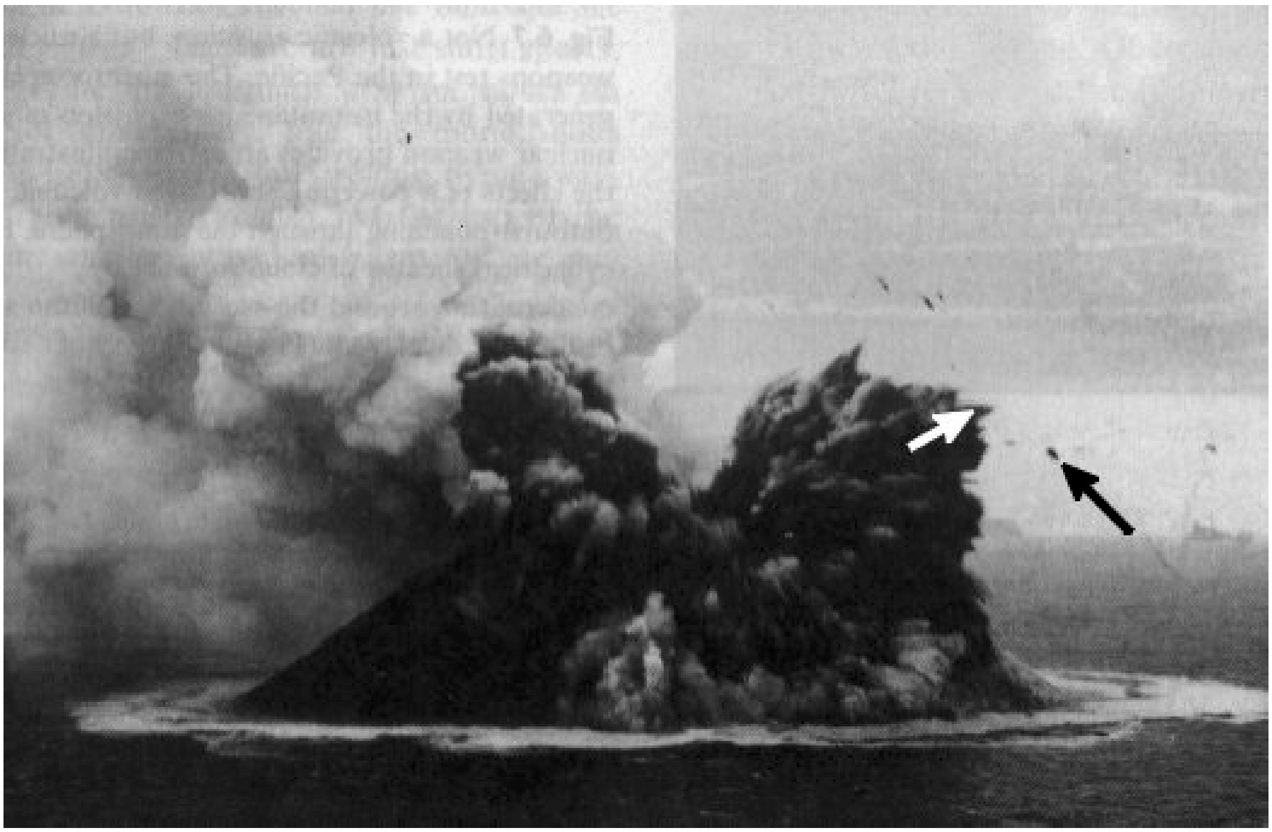

Figure 1. Eruption through a crater lake at Ruapehu Volcano, New Zealand, on September 24, 1995. The black cloud of tephra contains finger-like projections at its leading edge. Such projections tend to be headed by blocks or by concentrations of coarse debris whose collective air drag is less than that of surrounding tephra. Photograph by Ian Nairn, Institute for Geologic and Nuclear Sciences, New Zealand. [Use of this photo has been requested from the Institute for Geologic and Nuclear Sciences, New Zealand]

Governing Equations

Blocks in flight are subjected to forces (F) of drag against the ambient fluid, and of gravity. The acceleration of the block (*dv/dt)* in the horizontal (x) and vertical (z) directions results from those forces (fig. 2):

$$\frac{dv_x}{dt} = \frac{F_x}{m} = \frac{-v_x \rho_a v A C_d}{2m} \qquad \textbf{\textit{(1a)}}$$

$$\frac{dv_z}{dt} = \frac{F_z}{m} = \frac{-v_z \rho_a v A C_d}{2m} - g\frac{\rho_r - \rho_a}{\rho_r} \qquad \textbf{\textit{(1b)}}$$

where ρ_a is air density, $_r$ is density of the ballistic block, A is cross-sectional area of the block, C_d is drag coefficient on the block, m is the block's mass, g is gravitational acceleration (9.81 m/s^2), v is the block's velocity (i.e., the magnitude of the velocity vector) and t is time. The first term on the right-hand side of both equations is the force (per unit mass) of air drag. The second term on the right-hand side in equation ***1b*** represents the gravitational force of the block per unit mass. The effect of a horizontal tailwind of velocity w on block motion can be accommodated by replacing v_x with (v_x-w) in ***1a***, and v with $\sqrt{v_z^2 + (v_x - w)^2}$ in ***1a*** and ***1b***.

Equations ***1a*** and ***1b*** ignore the force associated with block rotation, known as the Magnus force, which is responsible for curve balls in baseball and (in part) for the stability of rifle-propelled bullets. In some cases the Magnus force may influence the trajectories of volcanically ejected blocks (Waitt and others, 1995). `Eject!` also ignores cross-wind forces, lift, variations in the gravitational-force vector with position, and the Coriolis force (the latter two are considered in long-range military ballistics calculations).

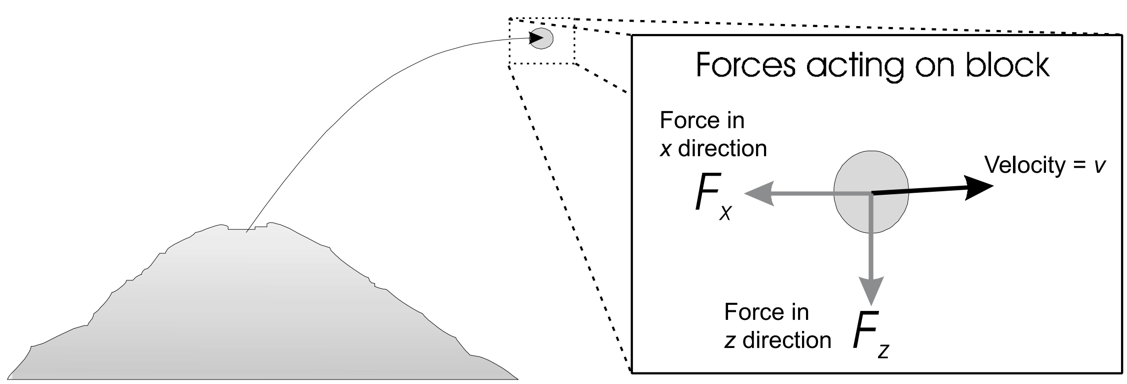

Figure 2. Illustration of the forces acting on a block ejected ballistically from a volcanic vent.

Calculating terms in the governing equations

The parameters in (1) are calculated as follows:

1. A and m depend on the shape and size of the block: $A = D^2$ for cubes (where D=block dimension), $(1/4)\,\pi D^2$ for spheres (where D=block diameter). $m = \rho_r D^3$ for cubes, $(1/6)\,\rho_r \pi D^3$ for spheres.
2. The air density (ρ_a) varies with elevation and is recalculated at each point in the trajectory using the equation for a perfect gas:

$$\rho_a = \frac{p}{RT} \qquad \textbf{(2)}$$

where T is in Kelvin, R is the gas constant for air (286.98 J/(kg K)), and p is the atmospheric pressure in Pascals. The pressure at a given elevation is calculated using the formula:

$$p = p_o \left(\frac{T}{T_o} \right)^{-g/\left(R(dT/dz)\right)} \qquad \textbf{(3)}$$

where p_o and T_o are the pressure (Pa) and temperature (Kelvin) at sea level and (dT/dz) is the thermal lapse rate in degrees Kelvin per meter[1]. Equation ***3*** is obtained by integrating the equation:

$$\frac{dp}{dz} = -\rho_a g = \frac{-p}{RT} g = \frac{-p}{R(T_o + (dT/dz)z)} g \qquad \textbf{(4)}$$

The pressure at sea level is assumed to be normal atmospheric, 1.013×10^5 Pa.

Values of T_o and *(dT/dz)* are given as input to the program.

Drag

The drag coefficient (C_d) is the drag force acting on a block, divided by the product of kinetic energy per unit volume of ambient fluid impinging on block and the block's cross-sectional area. The drag coefficient influences the final range of a block but varies greatly depending on its shape, orientation, and roughness. Experimental results (for example, Hoerner, 1965) indicate that C_d for variously shaped objects varies with two dimensionless parameters: Reynolds number (*Re*) and Mach number (*M*). The Reynolds number for external flow ($Re \equiv \rho_a v D/\eta$, where η is fluid

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[1] Because z increases upward, *dT/dz* should be negative if temperatures decrease with increasing altitude. This convention is assumed in equation ***3***. However `Eject!` assumes that positive values of thermal lapse rate, entered into the text box in program interface, imply decreasing temperature with elevation. Such values are converted to negative values within the code.

viscosity) relates the importance of viscous forces (in the denominator) to inertial forces (in the numerator). `Eject!` calculates the Reynolds number from density calculations (described above), specified input values of block diameter, velocity calculated by solution of equation ***1***, and gas viscosity, calculated from the empirical relation for air:

$$\eta = 0.0000172\left(\frac{390}{T+117}\right)\left(\frac{T}{273}\right)^{1.5} \qquad \textbf{\textit{(5)}}$$

where T is temperature in Kelvin, and viscosity is given in Pascal-seconds.

The value of C_d has been determined experimentally for a variety of shapes; (e.g., for spheres, fig. 3*A*). For blocks ejected during volcanic eruptions, Reynolds numbers almost always exceed ~1x10^3. For spheres at Re>~200 C_d is either nearly constant at ~0.5 (fig. 3*A*) or, at Re>~2x10^5, drops abruptly to about 0.1. The abrupt drop is associated with a shift in the boundary-layer separation point from a location upstream of the equator of the sphere to a point downstream of the equator. The Reynolds number at which C_d drops differs for differing block shape and block roughness; for spheres, `Eject!` assumes that C_d =~0.5 for 200<Re<2x10^5, and ~0.1 for Re>2x10^5. For cubes, `Eject!` does not consider reduced values of C_d because the Reynolds number at which boundary-layer separation occurs is so variable.

The relations above govern the drag coefficient so long as the Mach number of the block is less than about 0.5 (~175 m/s for air at T=25ºC, p=1 atm). At higher Mach numbers, the drag coefficient increases as pressure waves accumulate ahead of the block. At M=~1-2, pressure waves coalesce to a single shock wave and drag coefficients reach a maximum. At Mach numbers a few times greater than 1, C_d decreases as the shock front begins to wrap around the object.

`Eject!` calculates the Mach number ($M^{\gamma} \equiv v/c$, where v=velocity and c=sound speed in air) using the relation for sound speed in a perfect gas:

$$c = \sqrt{\gamma R T} \qquad \textbf{\textit{(6)}}$$

where γ is the ratio of specific heats (C_p/C_v) at constant pressure and constant volume (γ=1.4 for air). Using values of C_d versus M digitized from figures. 3*B*, 4*A*, and 4*B* for spheres and cubes, `Eject!` uses a cubic spline interpolation routine (Press and others, 1995, p. 101) to determine the drag coefficient for a given Mach number at M>0.5.

Reduced drag near vent

Prior to the early 1990’s, investigators assumed that blocks were ejected into still, ambient air from the moment they left the vent. In fact, visual observations (e.g., fig. 1) indicate that the blocks are initially enveloped in a cloud of tephra and other fragments that move at roughly the same velocity as the large blocks and therefore provide little or no drag (and may even provide some lift). The degree to which such drag is reduced and the distance over which drag is less than that in ambient air are not well constrained. Fagents and Wilson (1993) postulate that the ejected mass accelerates to a maximum velocity (v_o) at some distance (r_o) and time (t_o) from its initial position, then decelerates at the rate:

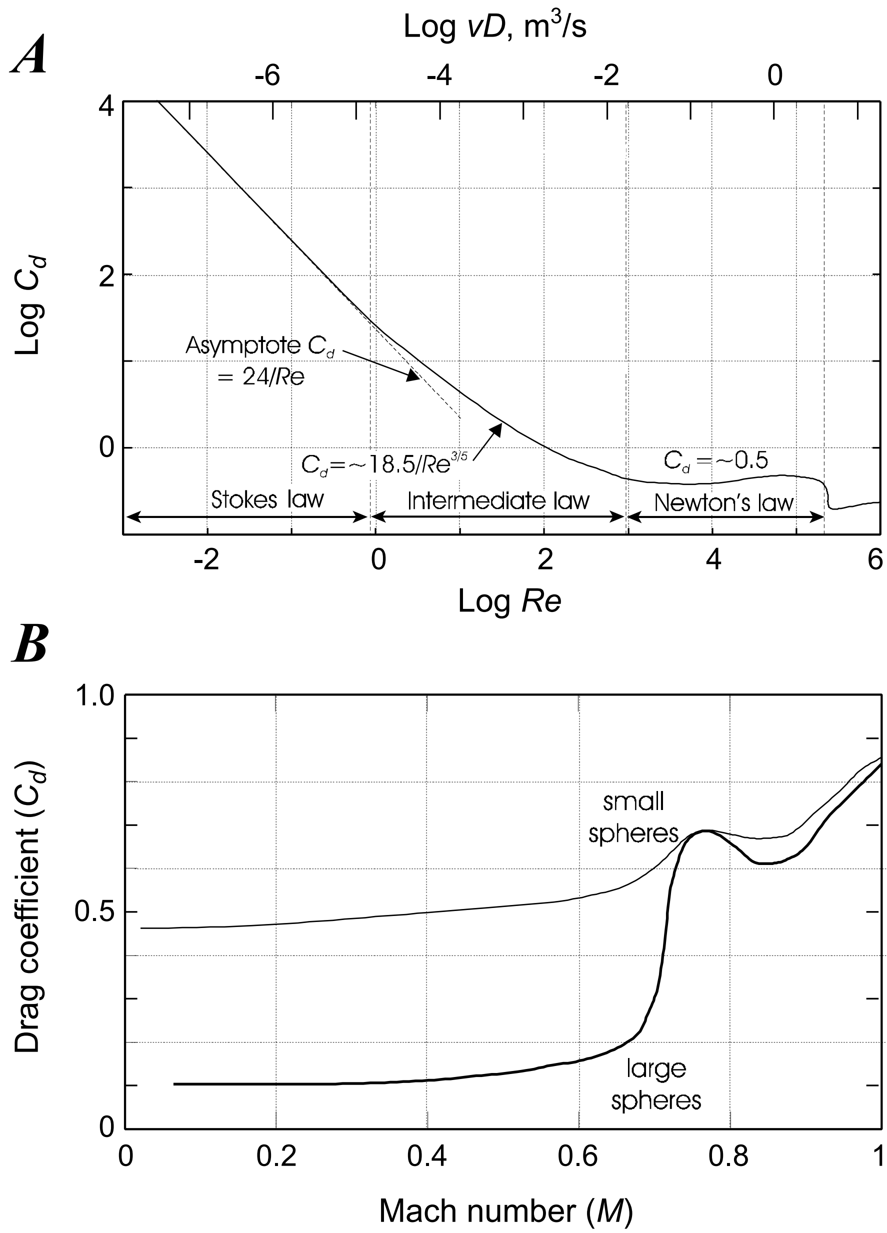

Figure 3. (***A***) Log drag coefficient (C_d) for spheres as a function of log Reynolds number, at low Mach number (from Bird and others, 1960, p. 192). Tick marks at top of plot give the product of velocity (*v*) times diameter (*D*) for spheres traveling in a fluid with the viscosity (1.8e-5 Pa s) and density (1.18 kg/m^3) of air at 25^o C, 1 atm pressure. (***B***) Drag coefficient for spheres in air at varying Mach numbers (from Hoerner, 1965). Top curve represents drag coefficients for spheres at Reynolds numbers below the critical transition (Re=~2x10^5) shown in figure 3*A*, whereas the lower curve represents drag coefficients for large spheres at *Re* above that transition.

$$v = v_o \left(\frac{r_o}{r} \right)^2 e^{-t/\tau} \quad \textbf{\textit{(7)}}$$

where the time constant t is related to the ratio of initial gas pressure (p_{gz}) to atmospheric pressure (p_a):

$$\tau = \frac{p_{gz}}{p_a} t_o \quad \textbf{\textit{(8)}}$$

The model Eject! does not estimate initial gas pressures because relations between initial pressure and velocity tend to be non-unique (they depend, for example, on initial temperature, mass fraction gas in the erupting mixture, the degree to which external water is involved, and the efficiency of conversion of gas enthalpy to kinetic energy; Wilson, 1980; Mastin, 1995). Instead, Eject! allows the user to specify (rather arbitrarily) a distance (r_d) from the vent over which drag is reduced. At a radial distance (r) less than r_d, a reduced drag coefficient (C_{dr}) is calculated from the formula:

$$C_{dr} = C_d \left(\frac{r}{r_d} \right)^2 \quad \textbf{\textit{(9)}}$$

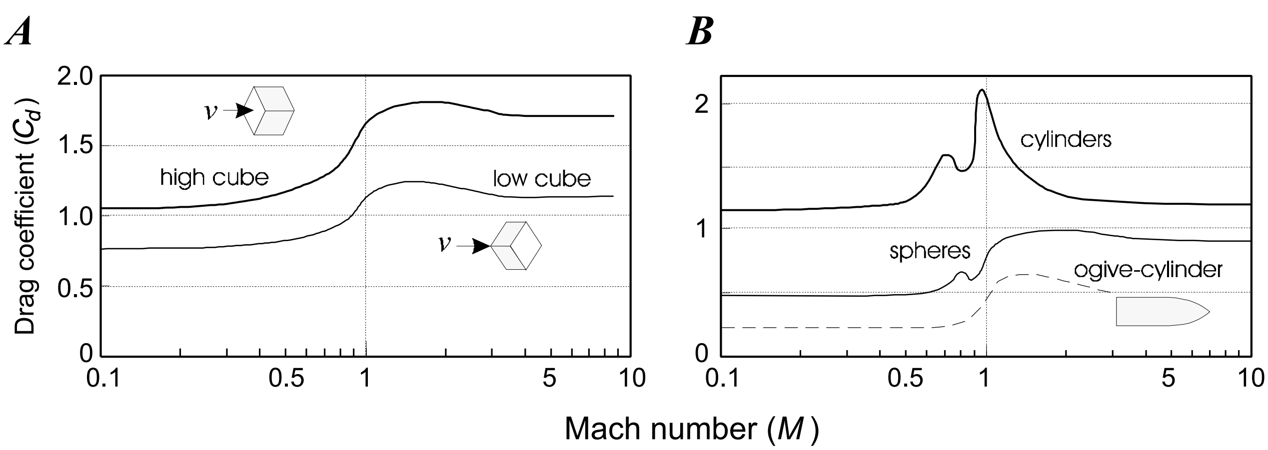

Figure 4. (***A***) Drag coefficient for cubes as a function of Mach number. The drag coefficient varies with orientation of the cube: C_d is highest when one face is oriented perpendicular to the direction of motion, lowest when the vertex between three faces is at the leading edge of the cube. (***B***) Drag coefficient as a function of Mach number for cylinders (top), spheres (middle), and the Ingalls-Mayevski ogive-cylinder (bottom). The last of these three is the shape of a G1 artillery shell for which classic ballistics tables were calculated (e.g., Ingalls, 1886). Figures derived from Hoerner (1965) and Miller (1979).

The value of r_d is likely to be tens to hundreds of meters, judging from photographs such as that in figure 1 (or those in Nairn and Self, 1978), and assuming that most of the reduced drag exists when the block is enveloped within the tephra cloud. More accurate estimates of r_d, and better relations between C_{dr} and C_d, will require more sophisticated modeling of volcanic explosions than has been accomplished to date.

Numerical solution

Equations ***1a*** aand ***1b*** above are integrated throughout the trajectory using a fourth-order Runge-Kutta method (Press and others, 1992, p. 706). The calculations continue until the vertical position (z) of the block reaches a specified landing elevation (z_f).

MODEL VALIDATION

The verification of numerically calculated ballistic trajectories such as these for blocks during volcanic eruptions has never been attempted. Such a study would help test the simplifying assumptions made in a model such as this one. In lieu of such a study, I compare the calculations made by this model with the special case of $C_d = 0$, for which analytical solutions exist, and with an independent set of ballistics calculations for small spheres (Miller, 1979). I also compare the results of `Eject!` with those for an approximate, analytical solution that assumes constant drag coefficient and constant air density.

Special case of zero air drag

In cases where the air drag is zero and where the density of ambient air is insignificant compared to that of the block, equations ***1a*** and ***1b*** simplify to $dv_x/dt = 0$, and $dv_z/dt = -g$, respectively. Integrating equations ***1a*** and ***1b*** twice with respect to time, the position of the block at a given point in its trajectory is then:

$$x(t) = v_i \cos\theta \, t \qquad \textbf{(10a)}$$

$$z(t) = v_i \sin\theta \, t - \frac{1}{2} g t^2 \qquad \textbf{(10b)}$$

where v_i is the initial velocity (i.e., the magnitude of the initial velocity vector). Assuming that the block travels to a landing point whose elevation equals that of the takeoff point, the final distance (x_{final}) reached by the block is:

$$x_{final} = \frac{v_i \sin 2\theta}{g} \qquad \textbf{(11)}$$

Block trajectories calculated using equations ***10a*** and ***10b*** are compared with those calculated by `Eject!`, assuming $C_d = 0$, in figure 5.

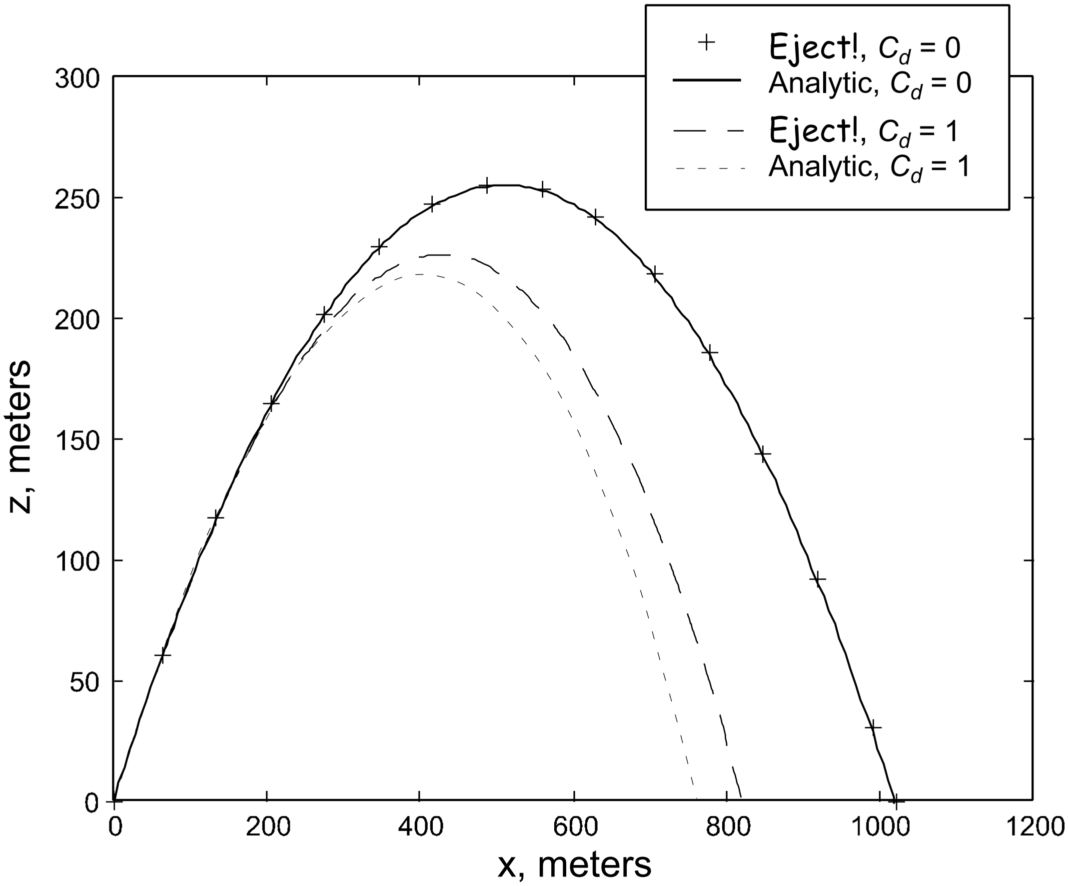

Figure 5. Comparison of analytical versus numerical calculations of ballistic trajectories under two sets of conditions, which are described in the text. The solid line and crosses represent the analytical solution and the numerical calculations, respectively, for $C_d = 0$. The dashed and dotted lines represent the numerical and analytical results, respectively, for the case of constant drag coefficient ($C_d = 1$) and air density (ρ_a=1.2 kg/m^3). Other input parameters for these runs are $D = 1$ m, $\rho_r = 2500$ kg/m^3, $\theta = 45°$, v_i=100 m/s, $w = 0$.

Comparison with other ballistics calculations

Table 1 compares the results of `Eject!` with calculations from Miller (1979) for small spheres. The trajectories of such small clasts are generally not studied by volcanologists because they are so easily influence by plume currents. On the other hand, their sensitivity to air drag makes them especially appropriate for a test of the numerical solution procedure used here. Miller conducted a set of experiments on drag of small spheres and used their results in a refined ballistics table for such spheres. Miller's calculations use the Sciacci approximation, which is used in classical ballistics tables (e.g., Ingalls, 1886, Hermann, 1930). The Sciacci approximation assumes that, for projectiles ejected in a near-horizontal trajectory, the term $\cos\theta$ can be replaced with a constant, making the integration simpler. No such approximation is used by `Eject!` The results of Miller's calculations differ from those of `Eject!` primarily in the third significant figure.

Table 1. Comparison of results calculated by `Eject!` (right three columns) with those from Miller (1979, table 2) for spheres 1.184 cm in diameter ejected with an initial velocity of 579 m/s at an angle of 0.8080 degrees above horizontal into air having a temperature of 15.3°C and a pressure of 1 atmosphere. The results give the final distance traveled (x_f), final velocity on impact (v_f), and time of flight (t_f).

		From Miller (1979)			From Eject!		
Material	Density (kg/m3)	x_f (m)	v_f (m/s)	t_f (sec)	x_f (m)	v_f (m/s)	t_f (sec)
pumice	700	45.63	37.09	0.456	48.59	37.34	0.476
Mg	1,740	87.63	61.78	0.638	86.90	59.77	0.64
basalt	2,700	117.90	78.70	0.737	117.20	77.11	0.728
steel	8,920	224.70	136.18	0.961	223.88	136.09	0.96
Pb	11,340	274.32	161.82	1.046	273.41	161.76	1.047
W	18,800	352.50	200.22	1.164	351.40	199.74	1.164

Comparison with analytical solution for constant drag coefficient and air density

In cases where the drag coefficient and air density are constant, analytical solutions for block trajectory and for the range traveled by blocks exist (Minakami, 1942; Self and others, 1980; Mastin, 1991). However in order to integrate equations ***1a*** and ***1b***, the product $v_x v$ in (***1a***) must be replaced with $v_x^2\mathrm{sgn}(v_x)$, and, in ***1b***, $v_z v$ must be replaced with $v_z^2\mathrm{sgn}(v_z)$, where "sgn" refers to the sign of these terms. This simplification results in underestimating the drag force along the trajectory by the factor $(\sin^4\theta + \cos^4\theta)^{1/2}$ (Sherwood, 1967). The resulting equations are:

$$\frac{dv_x}{dt} = \frac{-\rho_a C_d A v_x^2 \,\mathrm{sgn}(v_x)}{2m} \qquad \textbf{\textit{(12a)}}$$

$$\frac{dv_z}{dt} = \frac{-\rho_a C_d A v_z^2 \,\mathrm{sgn}(v_z)}{2m} - g \qquad \textbf{\textit{(12b)}}$$

These equations can then be integrated twice with respect to time, to give:

$$x(t) = \frac{1}{\mu}\ln\left(\mu v_i \cos\theta t + 1\right) \qquad \textbf{\textit{(13a)}}$$

$$z(t) = \frac{1}{\mu}\ln(\mu v_i \cos\theta t + 1) - \frac{1}{2}gt^2 \qquad \textbf{\textit{(13b)}}$$

where $\mu = (\rho_a C_d A)/2m$. The analytical solution for one particular case is shown by the dotted line in figure 5; the numerical solution calculated by `Eject!` for the same conditions is given by the dashed line in the same figure. Their lack of agreement results primarily from the modification of the governing equations required to produce an analytical solution.

USING Eject!

The program Eject!, along with this documentation, can be downloaded from the web site http://vulcan.wr.usgs.gov/Projects/Mastin. The program can operate on any Windows®-based personal computer running Windows 95 or later, and containing at least 3 Megabytes of hard disk space. To install the program, do the following:

1) Save the self-extracting Zip® file, "ejectzip.exe" to your hard drive,
2) Double-click on the file icon. It will expand into a dozen or more files and place them in a new folder entitled "Eject".
3) Go into the new folder "Eject", and double-click on the icon "setup.exe". Follow the instructions on the screen to install the program.
4) Go to the Start button, then to Programs, then choose the icon "Eject!" to launch the program.

Launching the program will display an introductory screen, followed (after clicking on the screen) by the window shown in figure 6. The box in the upper right hand corner gives specified input conditions for block velocity, ejection angle, tailwind velocity, and vertical distance of landing point below takeoff point. Also included are the elevation of the takeoff point, the atmospheric temperature at the vent, and the thermal lapse rate, all of which are used to calculate air density, pressure, and sound speed as a function of elevation during the block's trajectory.

The input conditions, and calculations of the block's trajectory during flight, may be written to a file whose name is specified in the "output options" box. If only a file name is given (excluding a full path name), the file will be written to the same directory that contains the program Eject! (usually "c:\program files\eject").

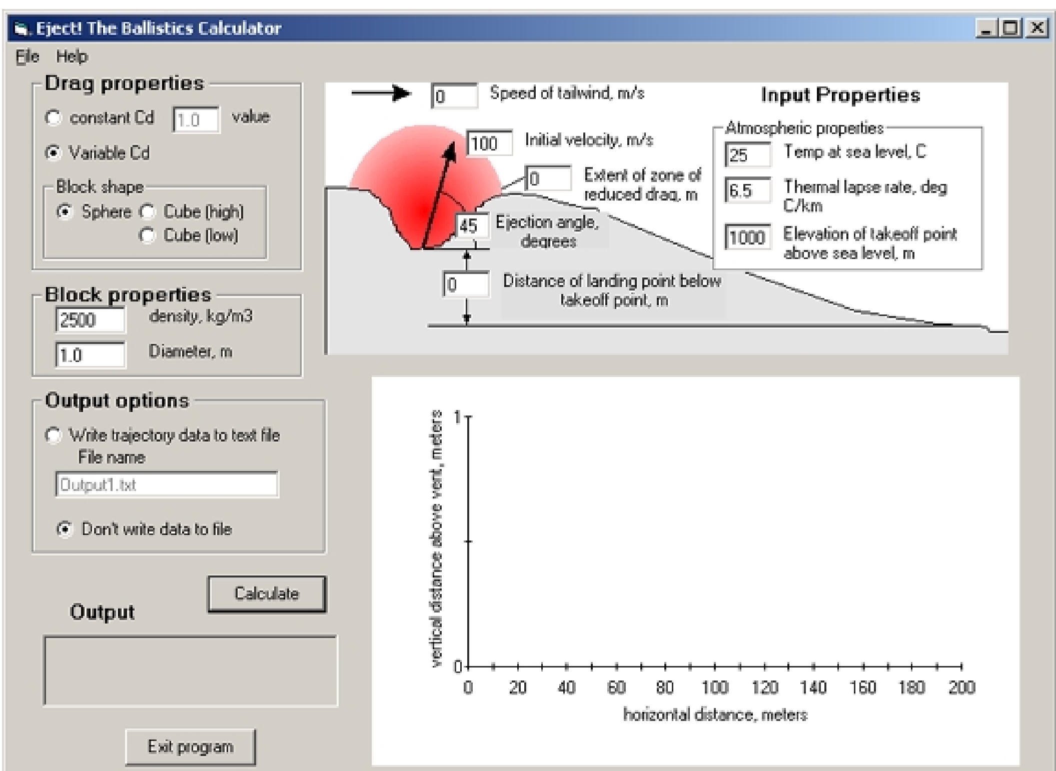

Figure 6. Graphical user interface for the program Eject!

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